VARIETIES OF TORI, STABILIZERS, AND RINGS OF INVARIANTS

In the structure theory of complex semisimple Lie algebras, the root space decomposition

$$\mathfrak{g} = \mathfrak{t} \oplus \bigoplus_{\alpha \in R} \mathfrak{g}_\alpha$$

relative to a maximal toral subalgebra $\mathfrak{t} \subseteq \mathfrak{g}$ plays a fundamental rôle. By conjugacy of such tori, this decomposition does not depend on the choice of $\mathfrak{t}$. For restricted Lie algebras $(\mathfrak{g}, [p])$ over an algebraically closed field of characteristic $p > 0$, this continues to hold, provided $\mathfrak{g} = \mathrm{Lie}(G)$ is associated to an algebraic group G. In the general case, maximal tori may not be conjugate.

Given a torus $\mathfrak{t} \subseteq \mathfrak{g}$ of maximal dimension $\mu(\mathfrak{g})$, we investigate the smooth variety $\mathcal{T}_\mathfrak{g}$ of injective homomorphisms $\mathfrak{t} \hookrightarrow \mathfrak{g}$. The canonical action of the automorphism group $\mathrm{Aut}_p(\mathfrak{t})$ on $\mathcal{T}_\mathfrak{g}$ induces a transitive action on the irreducible components of $\mathcal{T}_\mathfrak{g}$. One can therefore associate to $\mathfrak{g}$ the finite group $S(\mathfrak{g}) = \mathrm{Stab}_{\mathrm{Aut}_p(\mathfrak{g})}(\mathcal{X})$ of a component $\mathcal{X} \subseteq \mathcal{T}_\mathfrak{g}$. In this talk, I will discuss the connections between the structures of $\mathfrak{g}$, $S(\mathfrak{g})$ and $\mathcal{T}_\mathfrak{g}$. For Lie algebras of Cartan type, we consider the restriction map $k[\mathfrak{g}]^G \longrightarrow k[\mathfrak{t}]^{S(\mathfrak{g})}$, where G is the automorphism group of G.