

An old conjecture of Higman states that the number of different irreducible complex characters of the group  $U = U(n, q)$  of unitriangular matrices of degree  $n$  over the field with  $q$  elements should be a polynomial in  $q$  with integer coefficients. Classifying the irreducible characters of  $U$  is known to be a wild problem. The investigation of the ordinary representation theory of  $U$  therefore concentrated in the past few years on so called supercharacters, introduced by Yan, Andre, Isaacs, Diaconis and others. Supercharacters are classified and pairwise orthogonal. They are not irreducible in general, but every irreducible character of  $U$  occurs as constituent of precisely one supercharacter. This generalizes to the character theory of pattern subgroups of  $U$  which arise from closed subsets of the root system underlying  $U$ . In this talk I shall discuss methods relating supercharacters of  $U$  and those of some specially chosen pattern subgroups of  $U$ , which might lead to a prove of Higman's conjecture and some refinements hereof due to Lehrer and Isaacs.